Received: *January 17, 1998*, Accepted: *March 7, 1998*

Canonical equivalence between super D-string and type IIB superstring

Yuji Igarashi, Katsumi Itoh,[a*] **Kiyoshi Kamimura**[b] **and Rie Kuriki**[c]

Faculty of Education, Niigata University, Niigata 950-21, Japan;

[a] *Yukawa Institute for Theoretical Physics, Kyoto University Kyoto 606-01, Japan*

[b] *Department of Physics, Toho University, Funabashi 274, Japan*

[c] *Research Institute of Electrical Communication 2-1-1 Katahira, Aoba-ku, Sendai 980-8577, Japan*

Abstract: We show that the super D-string action is canonically equivalent to the type IIB superstring action with a world-sheet gauge field. Canonical transformation to the type IIB theory with dynamical tension is also constructed to establish the $SL(2, Z)$ covariance beyond the semi-classical approximations.

Keywords: Superstrings and Heterotic Strings, Branes in String Theory.

*On leave of absence from Faculty of Education, Niigata University, Niigata 950-21, Japan.

JHEP03(1998)002

Contents

1. Introduction

Much attention has been focused recently on effective action approach to D-branes, recognized as a key concept in understanding of string dualities [1]. The Dirac-Born-Infeld (DBI) actions have been considered to describe collective motions of the string solitons in the effective field theory [2, 3]. They have been constructed by the requirement of the conformal invariance [3], and their supersymmetric extensions with kappa symmetry have been given for allowed world volume dimensions [4, 5, 6].[1] Here these extended actions are referred to as D-brane actions.

In this paper we consider the $SL(2,Z)$ symmetry of the type IIB string theory [8].[2] In the effective action approach, the central machinery for discussing this symmetry is the "vector duality" transformations acting on world-volume gauge fields in the D-brane actions [2]. Several authors have argued that the transformed D-string (D1-brane) action takes an $SL(2,Z)$ covariant form, and the D3-brane action is invariant under the vector duality transformation [2, 9, 10]. These have been shown first for the bosonic sector, and then extension to the super D-branes has been given. Although the discussions provide us again with a support for the S duality, they share a drawback in common being based on semi-classical approximations. If the S duality is really a symmetry in string theories, it is natural to expect that one can establish, beyond semi-classical approximations, the $SL(2,Z)$ covariance of the D-string theory as well as that of the D3-action. It is the purpose of this paper to show that these desired properties of the D-branes are precisely described via canonical transformations. The idea that duality transformations can be essentially identified with canonical trans-

[1] The explicit form of the Wess-Zumino action for general (IIB) D-branes is given systematically in [7].

[2] In discussing the string duality, one has to specify the supergravity sector. As for supergravity backgrounds, we take the flat metric and include a dilaton and an axion backgrounds. Both of these are assumed to be constant in order not to spoil the kappa symmetry.

formations was already suggested in [11] for bosonic truncation of D3-brane and D-string. However the discussion was only made for the world-volume gauge fields and in the gauge fixed actions. We explore the whole structure of the canonical transformations. Actually, the canonical transformations we construct for D-string depend not only on the world-sheet gauge field but also on the string co-ordinates, especially the fermionic ones. We report here our results on D-strings only, because we found that the role of the canonical transformations are different, conceptually and technically as well, for D-strings and D3-branes. The D3-action will be investigated in a subsequent publication [12].

Our discussions are based on the canonical Hamiltonian formalism. The canonical Hamiltonian of a reparametrization invariant system is solely expressed by constraints, $\{\varphi_\alpha\}$. The Lagrange multipliers for first-class constraints are undetermined reflecting underlying gauge symmetries, while those for second-class ones are fixed by consistency conditions. The system is thus known to be purely determined by its constraint surfaces, $\varphi_\alpha = 0$. Suppose we have two such systems described by $\{\varphi_\alpha\}$ and $\{\tilde{\varphi}_\alpha\}$ and two sets of the constraints are related linearly: $\tilde{\varphi}_\alpha = M_\alpha{}^\beta \varphi_\beta$. If $M_\alpha{}^\beta$ is invertible, two systems may be considered to be equivalent, since they are defined by the same sets of the constraint surfaces.[3] We establish in this way equivalence between the D-string and the type IIB Green-Schwarz (GS) string systems. In these systems, we have first-class sets of the constraints corresponding to generators of the reparametrization, the kappa symmetry, and the U(1) gauge transformation. There appear also fermionic second-class constraints in relation to the kappa symmetry. The separation of the second-class set from the first-class one will not be discussed in detail, since it is not relevant to the canonical equivalence.

There are two main results to be reported here. First, the D-string is equivalent to the type IIB GS superstring. Second, the "vector duality" may be elevated to the canonical transformation without resort to any semi-classical approximations. The first claim is based on the observation that two actions have constraints related linearly and thus define the same constraint surfaces. The equivalent GS action should contain a contribution of the world-sheet gauge field, and a simple candidate for this is the "theta term". To discuss the second claim on the "vector duality", we perform another canonical transformation; the constraints of the D-string and the mapped constraints of the GS string share the same dynamical string tension which depends on the electric field. Once the integer quantization is assumed for the electric field, the tension is in the $SL(2,Z)$ covariant form [8] and the action which reproduces the remaining constraints on the string variables is the ordinary GS action but with the covariant tension. However, without the quantization, the action which generates the full constraints including those on the gauge field is not the GS type action with the tension

[3] In the extended phase space approach to constrained system, the BRST charges of such systems constructed directly from the first-class constraints are shown to be related via a canonical transformation acting also on ghost variables [13].

replaced by the dynamical one. We give an action yielding the full constraints: though it takes an unusual form, it is a supersymmetric extension of the DBI action.

2. Equivalence between D-string and type IIB string

We begin by considering the D-string action with kappa symmetry given in [9] (see also [14]). In addition to a world-sheet gauge field, it contains a dilaton ϕ and an axion χ backgrounds, which are assumed to be constants. The action is given by

$$S = -n \int d^2\sigma \left\{ e^{-\phi} \left[\sqrt{-G_F} + \epsilon^{\mu\nu} \Omega_{\mu\nu}(\tau_1) \right] + \frac{1}{2} \epsilon^{\mu\nu} \chi F_{\mu\nu} \right\} , \tag{2.1}$$

where

$$\begin{aligned} G_F &= \det(G_{\mu\nu} + \mathcal{F}_{\mu\nu}) = G + (\mathcal{F}_{01})^2, \quad & G &= \det G_{\mu\nu} = G_{00}G_{11} - (G_{01})^2 , \\ G_{\mu\nu} &= \Pi^m_\mu \Pi_{\nu,m} , & \Pi^m_\mu &= \partial_\mu X^m - \bar{\Theta}\Gamma^m \partial_\mu \Theta , \\ \mathcal{F}_{01} &= F_{01} - \epsilon^{\mu\nu}\Omega_{\mu\nu}(\tau_3), & F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu , \end{aligned}$$

and

$$\begin{aligned} \Omega_{\mu\nu}(\tau_1) &= \left\{ -\frac{1}{2}\bar{\Theta}\Gamma\tau_1\partial_\mu\Theta \cdot (\Pi_\nu + \frac{1}{2}\bar{\Theta}\Gamma\partial_\nu\Theta) \right\} - \{\mu \leftrightarrow \nu\} , \\ \Omega_{\mu\nu}(\tau_3) &= \left\{ -\frac{1}{2}\bar{\Theta}\Gamma\tau_3\partial_\mu\Theta \cdot (\Pi_\nu + \frac{1}{2}\bar{\Theta}\Gamma\partial_\nu\Theta) \right\} - \{\mu \leftrightarrow \nu\} . \end{aligned} \tag{2.2}$$

We have used the same conventions of the Dirac matrices and the $SL(2,R)$ description of N=2 SUSY in spinor indices as those given in [5].

Let $(X^m,\ P_m)$, $(\Theta_{A,\alpha},\ \{\mathit{\Pi}_\Theta\}_{A,\alpha})$, $(A_\mu,\ E^\mu)$ be canonically conjugate pairs of the phase space variables. The action (2.1) is described by a set of constraints

$$\varphi_0 = \frac{1}{2}[(\mathcal{P})^2 + (T_D)^2 G_{11}] \approx 0 , \tag{2.3}$$

$$\varphi_1 = \mathcal{P} \cdot \Pi_1 \approx 0 , \tag{2.4}$$

$$\psi = \mathit{\Pi}_\Theta + \bar{\Theta}\Gamma \cdot (\mathcal{P} - \Pi_1 \tau_D) - \frac{1}{2}(\bar{\Theta}\Gamma\Theta' \cdot \bar{\Theta}\Gamma\tau_D + \bar{\Theta}\Gamma\tau_D\Theta' \cdot \bar{\Theta}\Gamma) \approx 0 , \tag{2.5}$$

$$E^0 \approx 0 , \tag{2.6}$$

$$\partial_1 E \approx 0 , \tag{2.7}$$

where

$$\mathcal{P}^m = P^m + \bar{\Theta}\Gamma^m \tau_D \Theta' , \tag{2.8}$$

$E = E^1$ and $\Theta' = \partial_1\Theta$. Note that in the above set of the constraints the τ matrices acting on the $N = 2$ spinor indices of Θ_A appear only through the combination of τ_D defined by

$$\begin{aligned} \tau_D &= (E + n\chi)\tau_3 + n e^{-\phi}\tau_1 , \\ (\tau_D)^2 &= (T_D)^2 \mathbf{1}_2 , \end{aligned} \tag{2.9}$$

where

$$(T_D)^2 \;=\; (E+n\chi)^2+(ne^{-\phi})^2\,. \tag{2.10}$$

The string tension of the D-string may be identified with the co-efficient of G_{11} in the "Hamiltonian constraint" (2.3). It should be remarked that the D-string has the dynamical tension T_D, which is the desired form in view of the $SL(2,Z)$ covariance.[4] The matrix τ_D is a natural extension of that (defined as τ_E) given in [15]. One requires the constraints (2.3), (2.4), and a half of the fermionic one (2.5) to be first class, generators of the world-sheet reparametrization and the kappa symmetry transformation. The remaining half of (2.5) must be second-class constraints. This requirement is satisfied if the backgrounds ϕ and χ are constants. Canonical quantization of the D-string action (2.1) proceeds in parallel with that discussed in [15].

We establish first canonical equivalence of the D-string action (2.1) with type IIB GS string action with tension, $T=1$. It is based on an observation that if one replaces τ_D by τ_3 in (2.3), (2.4) and (2.5), one obtains the constraint set of type IIB superstring. This transformation on the τ matrix is realized by a SO(2) rotation around 2-axis [15],

$$U(\rho)=\exp\left(i\tau_2\frac{\rho}{2}\right), \tag{2.11}$$

with angle fixed by

$$\cos\rho=\frac{E+n\chi}{T_D}\,, \qquad\qquad \sin\rho=-\frac{ne^{-\phi}}{T_D}\,. \tag{2.12}$$

It leads to

$$U(\rho)^{-1}\;\tau_D\;U(\rho)=T_D\tau_3\,. \tag{2.13}$$

In order to get the constraint set of the GS string, we perform a canonical change of the phase space variables:

$$\begin{aligned}
x^m &= T_D^{1/2}\;X^m\,,\\
p^m &= T_D^{-1/2}\;P^m\,,\\
\theta &= T_D^{1/4}\;U(\rho)^{-1}\;\Theta\,,\\
\mathit{\Pi}_\theta &= T_D^{-1/4}\;\mathit{\Pi}_\Theta\;U(\rho)\\
\hat{A}_0 &= A_0\,,\\
\hat{A}_1 &= A_1+\left(\frac{E+n\chi}{2T_D^2}\right)P\cdot X+\mathit{\Pi}_\Theta\left[\left(\frac{E+n\chi}{4T_D^2}\right)-i\tau_2\left(\frac{ne^{-\phi}}{2T_D^2}\right)\right]\Theta\,,\\
\hat{E}^\mu &= E^\mu\,.
\end{aligned} \tag{2.14}$$

[4]The $SL(2,Z)$ covariant tension is $e^{\phi/2}T_D$. The difference came from our use of the string metric in the action; with the Einstein metric in it, one would obtain the covariant tension directly in the constraint.

These are obtained via a generating function

$$\hat{\mathcal{W}} = T_D^{1/2} p \cdot X + \hat{E}^\mu A_\mu + T_D^{1/4} \Pi_\theta U(\rho)^{-1}\, \Theta\,, \tag{2.15}$$

where T_D and $U(\rho)$ should be regarded as functions of $\hat{E}$ instead of E. When one replaces the D-string variables $(X,\ P,\ \Theta,\ \Pi_\Theta)$ by new ones $(x,\ p,\ \theta,\ \Pi_\theta)$ in the constraints (2.3), (2.4) and (2.5), there appear terms with derivatives $\partial_1 T_D$ and $\partial_1 U(\rho)$. All such terms, however, are proportional to the gauss law constraint, $\partial_1 E = E'$.

It is easy to show that the constraint set of the D-string is transformed into that of the GS string supplemented by the gauge field constraints,

$$\hat{\varphi}_0 = \frac{1}{2}[(\wp)^2 + g_{11}] \approx 0\,, \tag{2.16}$$
$$\hat{\varphi}_1 = \wp \cdot \pi_1 \approx 0\,, \tag{2.17}$$
$$\hat{\psi} = \Pi_\theta + \bar{\theta}\Gamma \cdot (\wp - \pi_1 \tau_3) - \frac{1}{2}(\bar{\theta}\Gamma\theta' \cdot \bar{\theta}\Gamma\tau_3 + \bar{\theta}\Gamma\tau_3\theta' \cdot \bar{\theta}\Gamma) \approx 0\,, \tag{2.18}$$
$$\hat{E}^0 \approx 0\,, \tag{2.19}$$
$$\partial_1 \hat{E} \approx 0\,, \tag{2.20}$$

where

$$\wp^m = p^m + \bar{\theta}\Gamma^m \tau_3 \theta'\,, \qquad \pi_1^m = x'^m - \bar{\theta}\Gamma^m\theta'\,, \qquad g_{11} = \pi_1 \cdot \pi_1\,. \tag{2.21}$$

One obtains the relations

$$\begin{aligned} \varphi_0 &\sim T_D \hat{\varphi}_0\,, \\ \varphi_1 &\sim \hat{\varphi}_1\,, \\ \psi &\sim T_D^{1/4}\ \hat{\psi}\ U(\rho)^{-1}\,, \end{aligned} \tag{2.22}$$

where $\sim$ represents equality up to the gauss law constraint. Therefore, the new canonical variables, $(x,\ p,\ \theta,\ \pi_\theta)$, turn out to be the GS string coordinates and their conjugates. In order to see the connection between the constrained systems of the D-string and GS string, we consider fermionic constraint algebra under the super-Poisson bracket:

$$\begin{aligned} \{\psi_{A,\alpha}, \psi_{B,\beta}\} &= 2(C\Xi)_{A,\alpha\ B,\beta}\,, \\ \{\hat{\psi}_{A,\alpha}, \hat{\psi}_{B,\beta}\} &= 2(C\hat{\Xi})_{A,\alpha\ B,\beta}\,, \end{aligned} \tag{2.23}$$

where C is the charge conjugation matrix in spinor space, and

$$\begin{aligned} \Xi_{AB} &= \not{P}\delta_{AB} - \not{\Pi}_1(\tau_D)_{AB}\,, \\ \hat{\Xi}_{AB} &= \not{\wp}\delta_{AB} - \not{\pi}_1(\tau_3)_{AB}\,. \end{aligned} \tag{2.24}$$

Note that both matrices are nilpotent, $\Xi^2 \sim \hat{\Xi}^2 \approx 0$. By multiplying Ξ, $\hat{\Xi}$ to ψ, $\hat{\psi}$ respectively, one may project out sets of first-class constraints, ψ_I and $\hat{\psi}_I$:

$$\begin{aligned} \psi_I &= \psi\Xi\,, \\ \hat{\psi}_I &= \hat{\psi}\hat{\Xi} = T_D^{-1/4}\ \psi_I\ U(\rho)\,. \end{aligned} \tag{2.25}$$

It turns out therefore that the bosonic and fermionic first-class constraints of the D-string are given by a linear combination of those of the GS string supplemented with U(1) gauge field. The fermionic second-class constraints of the D-string system are a mixture of the fermionic first-class and second-class constraints of the GS system. These linear combinations are clearly invertible. It follows then from a general theorem in canonical constrained systems [13] that the two systems with vanishing canonical Hamiltonian define the same constraint surfaces and therefore become canonically equivalent. The set of fermionic first-class constraints ψ_I in (2.25) is however reducible. Using projection operator $P_\pm = (1 \pm \tau_3)/2$, one obtains an irreducible set of the first-class constraints, $\psi \Xi P_+$. The orthogonal combination, $\psi_{II} = \psi P_-$, gives second-class constraints. It has been known [6, 16, 15] that this irreducible separation can be done for the D-string system without spoiling manifest Lorentz covariance. This is because that Ξ does not commute with $P_\pm$. On the other hand, since $\hat{\Xi}$ commutes with $P_\pm$, one inevitably looses either covariance or irreducibility in separation of first-class set from second-class one for the GS string system.

A candidate of GS string action which is canonically equivalent to (2.1) is given by

$$S = -\int d^2\sigma \left\{ \left[\sqrt{-g} + \epsilon^{\mu\nu}\omega_{\mu\nu}(\tau_3) \right] - \hat{E}\epsilon^{\mu\nu}\hat{F}_{\mu\nu} \right\} , \tag{2.26}$$

where $\omega_{\mu\nu}(\tau_3)$ is defined in terms of θ, and all other variables are defined also using the new canonical variables. $\hat{E}$ is an auxiliary field which becomes the conjugate momentum of $\hat{A}_1$. The "theta" term containing $\hat{E}$ is needed to generate the constraints for the gauge field. This action is shown to lead to the constraints (2.16) $\sim$ (2.20).

3. Canonical transformation and the S duality

In order to discuss $SL(2,Z)$ covariance, we may transform the D-string described by (2.1) into Type IIB string with the same dynamical tension T_D. It is achieved via another canonical transformation which only rotates the fermionic variables and gauge field.

$$\begin{aligned} \check{X}^m &= X^m , \\ \check{P}^m &= P^m , \\ \vartheta &= U(\rho)^{-1}\, \Theta , \\ \Pi_\vartheta &= \Pi_\Theta\, U(\rho) , \\ \check{A}_0 &= A_0 , \\ \check{A}_1 &= A_1 - i\frac{ne^{-\phi}}{2T_D^2}\Pi_\Theta \tau_2 \Theta , \\ \check{E}^\mu &= E^\mu . \end{aligned} \tag{3.1}$$

The generating function of the transformation reads

$$\check{\mathcal{W}} = \check{P}\cdot X + \check{E}^\mu A_\mu + \Pi_\vartheta U(\rho)^{-1}\, \Theta . \tag{3.2}$$

Using a matrix

$$\tau_F = \tau_3 \, T_D \,, \tag{3.3}$$

the constraints are shown to be transformed into

$$\check{\varphi}_0 = \frac{1}{2}[(\check{\mathcal{P}})^2 + T_D^2 \check{G}_{11}] \approx 0 \,, \tag{3.4}$$

$$\check{\varphi}_1 = \check{\mathcal{P}} \cdot \check{\Pi}_1 \approx 0 \,, \tag{3.5}$$

$$\check{\psi} = \mathit{\Pi}_\vartheta + \bar{\vartheta}\Gamma \cdot (\check{\mathcal{P}} - \check{\Pi}_1 \tau_F) - \frac{1}{2}(\bar{\vartheta}\Gamma\vartheta' \cdot \bar{\vartheta}\Gamma\tau_F + \bar{\vartheta}\Gamma\tau_F\vartheta' \cdot \bar{\vartheta}\Gamma) \approx 0 \,, \tag{3.6}$$

$$\check{E}^0 \approx 0 \,, \tag{3.7}$$

$$\partial_1 \check{E} \approx 0 \,, \tag{3.8}$$

where

$$\check{\mathcal{P}}^m = P^m + \bar{\vartheta}\Gamma^m \tau_F \vartheta' \,, \qquad \check{\Pi}_1^m = X'^m - \bar{\vartheta}\Gamma^m \vartheta' \,, \qquad \check{G}_{11} = \check{\Pi}_1 \cdot \check{\Pi}_1 \,. \tag{3.9}$$

The constraints (3.4) $\sim$ (3.8) are specified by τ_F, while those for the D-string (2.3) $\sim$ (2.7) by τ_D. Although they give rise to the same dynamical tension ${\tau_D}^2 = {\tau_F}^2 = T_D^2 \, \mathbf{1}_2$, they differ in the matrix structure which corresponds to the SO(2) rotation of the fermionic variables.

An action which generates the above set of the constraints reads

$$S = -n \int d^2\sigma \left\{ e^{-\phi} \sqrt{\left(\sqrt{-\check{G}} + \epsilon^{\mu\nu}\check{\Omega}_{\mu\nu}(\tau_3)\right)^2 - \left(\frac{1}{2}\epsilon^{\mu\nu}\check{F}_{\mu\nu}\right)^2} + \frac{1}{2}\chi\epsilon^{\mu\nu}\check{F}_{\mu\nu} \right\} . \tag{3.10}$$

It should be remarked that the bosonic truncation reduces this action to the conventional DBI action. Therefore, (3.10) gives a supersymmetric extension of the DBI action.

Comparing (3.4) $\sim$ (3.8) with (2.16) $\sim$ (2.20), one recognizes that the above constrained system describes the GS string with dynamical tension T_D. The tension becomes the $SL(2,Z)$ covariant form, if the electric field is quantized to be integer values. Quantization of U(1) gauge field can be done, compactifying the world-sheet co-ordinate σ into a circle (of length 2π). One takes the temporary gauge, $\check{A}_0 = 0$, leaving $\check{A}_1$ subjected to the periodic boundary condition, $\check{A}_1(\tau,\sigma) = \check{A}_1(\tau,\sigma+2\pi)$. A large gauge transformation, $\check{A}_1 \to \check{A}_1 + m$ with integer m, is known to only acts as a constant shift for the zero mode,

$$\check{\mathcal{A}}_1 = \int d\sigma \check{A}_1 \to \check{\mathcal{A}}_1 + 2\pi m \,. \tag{3.11}$$

If one identifies the zero mode configurations connected mutually via the large gauge transformations, $\check{\mathcal{A}}_1$ becomes an angular variable. The conjugate zero mode momentum $\check{\mathcal{E}}^1$, therefore, is quantized to be an integer value m. The Hamiltonian of the system is a function of the electric field $\check{E}^1$, and diagonalized in the momentum representation. Clearly, only the $\check{\mathcal{E}}^1$ dependence can survive in the physical state through the gauss-law constraint,

$$\partial_1 \check{E}^1 | \, phys >= 0 \,. \tag{3.12}$$

This gives the integer quantization of the electric field, generating NS-NS charge, $\check{E}^1 = m$.

Although the constrained system with (3.4) $\sim$ (3.8) describes manifestly the $SL(2,Z)$ covariance of the D-string theory, one can not immediately see the covariance in the action (3.10). If one performs, however, the electric field quantization described above first, one is left with the constrained system corresponding to the action:

$$S = -\int d^2\sigma \left\{ \sqrt{(m+n\chi)^2 + (ne^{-\phi})^2} \left[\sqrt{-\check{G}} + \epsilon^{\mu\nu}\check{\Omega}_{\mu\nu}(\tau_3) \right] \right\} . \qquad (3.13)$$

We emphasize that this form of action may be written only after the canonical transformation; the crucial element is the SO(2) rotation (3.2) which changes the τ-matrix structure from τ_D to τ_F.

4. Discussion

We have constructed two sets of the canonical transformations: the first one directly exhibits the equivalence of the D-string with the type IIB string with the normal tension; the second one is given to replace the vector-duality transformation. In both cases, the transformations in fermionic sector contain an SO(2) rotation around 2 axis of N=2 spinor space. The approach given here is not based on any semi-classical approximations, and the results precisely agree with the $SL(2,Z)$ covariance of D-string. We close this paper with the following remarks.

1. If one wishes to show the equivalence between two theories more explicitly, one may employ BRST quantization scheme. After performing the canonical transformation between the string variables, one eliminates the fermionic second-class constraints, leaving first-class set of the constraints for each system. One is given by a linear combination of the other. Since this relation is clearly invertible as long as $E \neq 0$, two BRST charges constructed from different sets of the constraints are related by a canonical transformation defined in extended phase space, as a result of the general theorem given in [13]. Construction of the BRST charges is tedious but important to compare two theories.[5] In suitable gauge choices, one may discuss quantization of the D-string system and the GS string system, and examine their spectrum as well as the structure of the global supersymmetry. A crucial difference between two systems is that the D-string system can be covariantly quantized with finite ghost variables, unlike the GS system.

2. We have restricted ourselves to the flat target-space metric and constant backgrounds of the dilaton and axion fields. In addition, the other backgrounds such as anti-symmetric tensor fields have been ignored here. These restrictions should be removed to establish the S duality.

[5] The on-shell nilpotent BRST charge for the super D-string is given in [15].

JHEP03(1998)002

3. There have been some attempts to construct manifestly $SL(2,Z)$ covariant superstrings where additional world-sheet gauge field is introduced to respect the $SL(2,Z)$ symmetry more explicitly [17]. It will be possible to give an extended canonical transformation, which connects such actions with some extension of the GS action with a couple of world-sheet gauge fields.

4. It is important to examine if our canonical transformation approach given here for the D-string applies to the D3-brane and other brane actions in type IIB theory. As will be reported in a subsequent publication [12], canonical transformation can be construnted to show the $SL(2,Z)$ invarinace of the D3-brane action. For even p-branes such as D2-brane and D4-brane, investigations of the relationship between the type IIA theory and a compactified $D = 11$ M theory based on canonical transformation will be especially interesting.

Acknowledgments

K.I. would like to thank to YITP for its kind hospitality extended to him. R.K. is obliged to A.Sugamoto for enjoyable conversations on dualities in string theories and for encouragement. K. K. thanks Joaquim Gomis and Machiko Hatsuda for valuable discussions.

References

[1] J. Polchinski, *Phys. Rev. Lett.* **75** (1995) 4724 [hep-th/9510017]; *TASI Lectures on D-Branes*, hep-th/9611050.

[2] A.A. Tseytlin, *Self-duality of BI action and D3-brane of type IIB superstring theory*, *Nucl. Phys.* **B 469** (1996) 51 [hep-th/9602064].

[3] R.G. Leigh, *Dirac-Born-Infeld action from Dirichlet σ-model*, *Mod. Phys. Lett.* **A 4** (1989) 2767;
C. Callan, C. Nappi, C. Lovelace and S. Yost, *String loop corrections to beta functions*, *Nucl. Phys.* **B 288** (1987) 525;
C. Callan, C. Nappi, C. Lovelace and S. Yost, *Loop corrections to superstring equations of motion*, *Nucl. Phys.* **B 308** (1988) 221;
C. Schmidhuber, *D-brane actions*, *Nucl. Phys.* **B 467** (1996) 146 [hep-th/9601003].

[4] M. Cederwall, A. von Gussich, B.E.W. Nilsson and A. Westerberg, *The Dirichlet super three brane in ten-dimensional type IIB supergravity*, *Nucl. Phys.* **B 490** (1997) 163 [hep-th/9610148];
M. Cederwall, A. von Gussich, B.E.W. Nilsson, P. Sundell and A. Westerberg, *The Dirichlet super p-branes in ten-dimensional type IIA and IIB supergravity*, *Nucl. Phys.* **B 490** (1997) 179 [hep-th/9611159].

[5] P.K. Townsend, *D-branes from M-branes*, *Phys. Lett.* **B 373** (1996) 68 [hep-th/9512062];
M. Aganagic, C. Popescu and J.H. Schwarz, *D-brane actions with local kappa symmetry*, *Phys. Lett.* **B 393** (1997) 311 [hep-th/9610249].

[6] M. Aganagic, C. Popescu and J.H. Schwarz, *Gauge invariant and gauge fixed D-brane actions*, *Nucl. Phys.* **B 495** (1997) 99, hep-th/9612080.

[7] K. Kamimura and M. Hatsuda, *Canonical formalism for IIB D-branes*, TOHO-FP-9757, hep-th/9712068.

[8] J.H. Schwarz, *Phys. Lett.* **B 360** (1995) 13 [hep-th/9508143]; Erratum, *ibid.* **364** (1995) 252.

[9] M. Aganagic, C. Popescu and J.H. Schwarz, *Dual D-brane actions*, *Nucl. Phys.* **B 496** (1997) 215 [hep-th/9702133].

[10] S.P. de Alwis and K. Sato, *D strings and F strings from string loops*, *Phys. Rev.* **D 53** (1996) 7187 [hep-th/9601167];
C. Schmidhuber, *D-brane actions*, *Nucl. Phys.* **B 467** (1996) 146 [hep-th/9601003].

[11] Y. Lozano, *D-brane dualities as canonical transformations*, *Phys. Lett.* **B 399** (1997) 233 [hep-th/9701186].

[12] Y. Igarashi, K. Itoh and K. Kamimura, in preparation.

[13] M. Henneaux and C. Teitelboim, *Quantization of Gauge Systems*, Princeton University Press, 1992.

[14] E. Witten, *Bound states of strings and p-branes* *Nucl. Phys.* **B 460** (1995) 335 [hep-th/9510135].

[15] M. Hatsuda and K. Kamumura, *Covariant quantization of the Super-D-string* Toho-FP-9756, hep-th/9708001.

[16] R. Kallosh, *Covariant quantization of D-branes*, *Phys. Rev.* **D 56** (1997) 3515 [hep-th/9705056].

[17] P.K. Townsend, *Membrane tension and manifestly IIB S duality*, *Phys. Lett.* **B 409** (1997) 131 [hep-th/9705160];
M. Cederwall and P.K. Townsend , *The manifestly $SL(2,Z)$ covariant superstring*, *JHEP* **09** (1997) 003 [hep-th/9709002];
M. Cederwall and A. Westerberg, *World-volume fields, $SL(2,Z)$ and duality*, *JHEP* **02** (1998) 004 [hep-th/9710007].